

On the Ter-Mikaelian and Landau–Pomeranchuk effects for induced soft gluon radiation in a QCD medium

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Abstract

The polarization of a surrounding QCD medium modifies the induced gluon radiation spectrum of a high-energy parton at small transverse momenta for a single interaction and for multiple scatterings as well. This effect is an analogue of the Ter-Mikaelian effect in QED, superimposed to the Landau–Pomeranchuk effect, however it appears in QCD in a different phase space region.

Introduction: The soft gluon radiation induced by energetic partons propagating through a medium of quarks and gluons is of great interest now, since the radiative energy loss and corresponding stopping power of quark or gluon jets might serve as a probe of the quark-gluon plasma formation in ultrarelativistic heavy-ion collisions (cf. [1, 2] and references therein). It was recently shown [2, 3, 4] that the Landau–Pomeranchuk–Migdal (LPM) effect (cf. [5]) plays a very important role for the formation of gluon bremsstrahlung in a QCD medium. In analogy with QED the LPM effect for gluon radiation is related to a destructive interference between radiation amplitudes due to multiple scatterings of a high-energy color charge propagating through the medium. The crucial physical characteristics here is the radiation formation length l_f . To radiate coherently the emitting particle must be undisturbed while traversing the formation length. Multiple scatterings within l_f can cause a loss of coherence. This leads to a suppression and qualitative change of the soft radiation spectrum compared to the well-known Bethe–Heitler formula, where the spectrum is simply additive in the number of scatterings. It should be stressed that the LPM effect becomes operative in a rather restricted kinematical region of the energy E of the radiating particle and energy ω of the radiation: the formation length must be large in comparison with the mean free path λ , $l_f \gg \lambda$. In particular, to fulfill this condition in the case of an extended medium, modeled by screened static scattering centers [2], the relations $\omega \ll E^2/(\lambda\mu^2)$ within QED or $\omega \gg \lambda\mu^2$ within QCD must be fulfilled [3] (μ is the Debye screening

mass).

At the same time it is well-known from electrodynamics that the formation length can be modified not only by multiple scatterings of the radiating particle but also by the medium polarization, i.e. the emitted photon is affected. It was firstly pointed out by Ter-Mikaelian [6, 7] that the dielectric polarization of the medium can also cause a loss of coherence, suppressing in this way the emission process. This effect, also known as dielectrical effect or longitudinal density effect, suppresses the very soft bremsstrahlung photons, while the LPM effect becomes operative at larger photon energies.

The strong reduction of the formation length, reflecting the suppression of radiation due to the dielectrical effect, can be seen in electrodynamics by the following simple qualitative estimates. In the high-frequency approximation the dielectrical "constant" becomes $\epsilon = 1 - \omega_p^2/\omega^2$, where ω_p is the plasma frequency. In a medium the dispersion relation between energy ω and momentum \vec{k} becomes $\omega = |\vec{k}|/\sqrt{\epsilon}$ in contrast to the vacuum dispersion relation $\omega = |\vec{k}|$ (we use units with $\hbar = c = 1$). As a result the "vacuum" formation length $l_f = 2\gamma^2/\omega$ is reduced to $l_f = 2\omega/\omega_p^2$ in the interval $\omega_p \ll \omega \ll \gamma\omega_p$, where γ is the Lorentz factor of the radiating particle.

The Ter-Mikaelian (TM) effect is important in electrodynamics because it cuts off the soft bremsstrahlung spectrum at low energies, thus removing the infrared divergence. The recent experiments with high-energy electrons at SLAC [8] confirmed the TM effect, predicted 45 years ago.

Due to the long-range properties of color forces one can expect that the polarization of a QCD medium is also important for the induced gluon radiation. A throughout consideration of this problem could be based on the gluon polarization tensor in a hot medium [9, 10] together with proper modifications of the propagators and vertices in the gluon radiation Feynman diagrams. However, even for the Abelian theory such a consideration appears rather involved [5], and a simpler approach provides better insight in the problem [7]. Fortunately, in the high-temperature limit the needed dispersion relation for gluons in a QCD medium [10] can be approximated by introducing an effective gluon mass ω_0 [11, 12, 13], which depends on the temperature T (we consider a charge-symmetric medium). Within this approximation the dispersion relation for gluons has a form similar to the Abelian case [14]

$$k^2 \equiv k^\mu k_\mu = \omega^2 - k_\parallel^2 - k_\perp^2 = \omega_0^2, \quad (1)$$

where $k^\mu = (\omega, k_\parallel, k_\perp)$ is the gluon four-momentum, and $\omega_0(T)$ parameterizes the temperature dependent gluon self-energy. Such an effective quasi-particle model of massive gluons is proven successful in reproducing the results of lattice calculations for various collective properties of the hot QCD medium, such as the equation of state and Debye screening mass [12]. Moreover, a detailed analysis [12, 13] of the recent QCD

lattice data [15] allows to extract $\omega_0(T)$. Typically ω_0 is in the order of a few hundred MeV already slightly above the confinement temperature.

Here we are going to consider the influence of the QCD medium polarization on the induced soft gluon emission and demonstrate the existence of the non-Abelian analogue to the TM effect. Basing on the gluon dispersion relation (1) we begin treating the gluon radiation for single scattering of a fast parton and show that, due to the polarization of the surrounding medium, the resulting gluon spectrum in the small transverse momentum region is considerably suppressed and the infrared divergence at $k_\perp \rightarrow 0$ is regularized by the dimension parameter ω_0 . Then this result is extended to multiple scatterings in a QCD medium within the potential model [2]. In the region of small transverse momenta of the radiated gluons, which is relevant for the TM effect, we analyze the medium corrections for this specific QCD radiation which is actually absent in the Abelian case. Another influence of the medium on the radiation is the multiple gluon scattering due to the non-Abelian interaction considered in ref. [3]. Here we focus on single-gluon emission; multiple gluon emission is dealt with in ref. [16].

Single scattering: To demonstrate the importance of the modified dispersion relation eq. (1) we consider first the induced gluon radiation for a single quark-quark scattering. The cross section for this process was originally derived by Gunion and Bertsch [17]. Following their calculations we use the light cone representation of four-vectors and the $A^+ = 0$ gauge for the gluon field. The variables for the radiation process are defined in fig. 1. In the center-of-mass frame of the colliding quarks the initial momenta of quarks and the final gluon and its polarization are

$$p_i = [\sqrt{s}, 0, \vec{0}_\perp], \quad p'_i = [0, \sqrt{s}, \vec{0}_\perp], \quad (2)$$

$$k = [x\sqrt{s}, (k_\perp^2 + \omega_0^2)/(x\sqrt{s}), \vec{k}_\perp], \quad \epsilon = [0, 2\vec{k}_\perp \vec{\epsilon}_\perp / (x\sqrt{s}), \vec{\epsilon}_\perp], \quad (3)$$

where $x = (\omega + k_\parallel)/\sqrt{s}$ and $s = (p_i + p'_i)^2$. The components of the gluon polarization obey the condition $\epsilon k = 0$ which is valid for massive bosons too [14]. Using the condition that the quarks after scattering are on-mass shell one derives for $x \ll 1$ the momentum transfer as

$$q = \left[-\frac{q_\perp^2}{\sqrt{s}}, \frac{x^{-1}k_\perp^2 + q_\perp^2 - 2\vec{q}_\perp \vec{k}_\perp}{(1-x)\sqrt{s}} + \frac{\omega_0^2}{x\sqrt{s}}, \vec{q}_\perp \right]. \quad (4)$$

As shown in ref. [17] in the limit of small x only the set of diagrams plotted in fig. 1 is important in the given gauge. (To construct a gauge invariant amplitude one should take into account the whole set of diagrams including the radiation from the target quarks [2, 3] and also keep for the emitted and exchanged gluons the same mass parameter ω_0 .) We take into account only the leading terms to the amplitude which are proportional to s . A straightforward calculation results then in the amplitudes

corresponding to emission from the projectile quark line

$$M_a^{\text{rad}} = M^{\text{el}} 2g \frac{\vec{\epsilon}(\vec{k}_\perp - x\vec{q}_\perp)(1-x)}{(x\vec{q}_\perp - \vec{k}_\perp)^2 + (1-x)\omega_0^2} C_a, \quad (5)$$

$$M_b^{\text{rad}} = -M^{\text{el}} 2g \frac{\vec{\epsilon}\vec{k}_\perp(1-x)}{\vec{k}_\perp^2 + \omega_0^2} C_b, \quad (6)$$

where $C_{a,b}$ are color matrix elements associated with diagrams 1a and 1b divided by the color factor C^{el} of the elastic scattering amplitude. The elastic scattering amplitude is $M_{\text{el}} = -2sq_\perp^{-2}g^2C^{\text{el}}$. The leading term of the amplitude corresponding to the radiation from the gluon line via the triple vertex is not modified, i.e. it remains the same as in case of radiation in the vacuum, $M_c^{\text{rad}} = M^{\text{el}} 2g \frac{\vec{\epsilon}(\vec{q}_\perp - \vec{k}_\perp)(1-x)}{(\vec{k}_\perp - \vec{q}_\perp)^2} C_c$, where for the color factor $C_c = C_a - C_b$ holds.

In the limit of small values of x , where $xq_\perp \ll k_\perp$, the sum of the amplitudes yields

$$M_1^{\text{rad}} = M^{\text{el}} 2g\vec{\epsilon}_\perp \left[\frac{\vec{k}_\perp}{k_\perp^2 + \omega_0^2} + \frac{\vec{q}_\perp - \vec{k}_\perp}{(\vec{k}_\perp - \vec{q}_\perp)^2} \right] C_c. \quad (7)$$

In the Abelian case $C_{a,b} = 1$, and M_a^{rad} and M_b^{rad} cancel. This fact is well-known in electrodynamics: soft photon radiation of relativistic electrons is strongly focused along the particle's velocity, i.e. bremsstrahlung is suppressed as $1/s$ in the central rapidity region where $x \rightarrow k_\perp/\sqrt{s}$. As seen in eq. (7) this specific non-Abelian radiation is modified by the medium polarization in the small- k_\perp region where $k_\perp \ll \omega_0$.

The r.h.s of eq. (7) is connected with the radiation amplitude $R_1 = M_1^{\text{rad}}/(M^{\text{el}}C^{\text{el}})$ which, after squaring and averaging/summing over initial/final color and polarization states, gives the multiplicity distribution of emitted gluons $dn^{(1)}/d^2k_\perp dy = \overline{|R_1|^2}$ (here y is the gluon rapidity)

$$\overline{|R_1|^2} = \frac{C_1}{\omega_0^2} \frac{\omega_0^4 + 2\omega_0^2\vec{k}_\perp\vec{q}_\perp + k_\perp^2 q_\perp^2}{(\omega_0^2 + k_\perp^2)^2 (\vec{k}_\perp - \vec{q}_\perp)^2}, \quad (8)$$

where $C_1 = 4\pi\alpha_s \frac{N^2-1}{N}$ for N colors; $\alpha_s = g^2/(4\pi)$.

In the limit $k_\perp \rightarrow 0$ the spectrum (8) is finite, i.e. the infrared divergence is removed due to the modified dispersion relation eq. (1). In the region $1 \ll (\omega_0/k_\perp)^2 \ll q_\perp/k_\perp$ the three-gluon vertex contribution (diagram 1c) can be neglected compared to the leading part from the quark-gluon vertex (diagrams 1a,b). We use this result later on for the analysis of the polarization effects in the multiple scattering case. In fig. 2, various contributions $\overline{|R_1^i|^2}$ to the radiation amplitude $\overline{|R_1|^2}$ are displayed as a function of k_\perp (in units of ω_0) for a given momentum transfer to show the influence of the modified dispersion relation. As can be seen in fig. 2 the polarization effect cuts off the gluon spectrum at small values of k_\perp . In contrast to the vacuum case ($\omega_0 = 0$) for

given q_\perp , in the limit $k_\perp \rightarrow 0$ the dominant contribution comes from the three-gluon vertex diagram.

Double scattering: In order to consider the polarization effect in the multiple scattering we employ the potential model [2] with static scattering centers. The center located at \vec{x}_j creates a screened Coulomb potential $V^a(\vec{q}) = gT^a \exp\{-i\vec{q}\vec{x}_j\}/(\vec{q}^2 + \mu^2)$, where μ is the color screening mass, T^a denotes a generator of SU(N), and \vec{q} is the spatial momentum transfer vector. The distance between two consecutive scatterings, λ , is assumed to be large compared to the screening length, i.e. $\lambda\mu \gg 1$. As mentioned above and considered in more detail in ref. [2], in the $A^+ = 0$ gauge the gluon radiation off the target quarks is suppressed in the kinematical region $\omega \gg k_\perp$, while x is still small, i.e. $x \ll 1$. Within the framework of the static scattering center model the condition $\omega \gg k_\perp$ is associated with high rapidity of emitted gluons. Below we focus just on the high-rapidity, soft radiation satisfying $E \gg \omega \gg k_\perp$. It is remarkable that within such a condition the radiation spectrum for single scattering off a static source [2] looks the same as the spectrum eq. (8) which is obtained for colliding quarks and at midrapidities.

Let us now consider two static potentials separated by the distance L with $L\mu \gg 1$. Similar to the case of a single scattering there is an interval of small values of k_\perp where the radiation contribution from the internal gluon lines according to triple gluon vertices and the diagram with four-gluon vertex [3] can be neglected. Basing on the momentum dependence of the corresponding amplitudes elaborated in ref. [3] one finds the conditions $k_\perp \ll q_{\perp 1,2}$, $\vec{k}_\perp \ll \vec{q}_{\perp 1} + \vec{q}_{\perp 2}$, where $\vec{q}_{\perp 1,2}$ denote the momentum transfers at scattering centers 1, 2. Due to the dispersion relation (1) we obtain the additional restriction $1 \ll \left(\frac{\omega_0}{k_\perp}\right)^2 \ll \frac{q_{\perp 1,2}}{k_\perp}$. In the given interval only radiation from quarks lines (cf. fig. 3) is important. At the same time it is just the radiation from the quark lines which is affected by the modified dispersion relation $k^2 = \omega_0^2$. Since we are going to demonstrate the apparency of the polarization effect for the induced radiation we simplify the analysis and restrict ourselves to the kinematical region where the radiation from quark lines dominates.

Under the conditions $L\mu \gg 1$ and $E \gg \omega \gg q_{\perp 1,2}$ the factorization of the total amplitude into an elastic part and a radiation part R_2 is straightforward and yields for the set of diagrams displayed in fig. 3

$$R_2 = 2g \frac{\vec{\epsilon}_\perp \vec{k}_\perp}{k_\perp^2 + \omega_0^2} \left\{ T^{a2} [T^{a1}, T^b] \exp\{ikx_1\} + [T^{a2}, T^b] T^{a1} \exp\{ikx_2\} \right\} T^{a1} T^{a2}, \quad (9)$$

where $x_1 = (0, \vec{x}_1)$ and $x_2 = (t_2, \vec{x}_2)$ are the four-coordinates of two potentials with $t_2 = (z_2 - z_1)/v_z = L/v_z$ and v_z as the longitudinal velocity of the high-energy parton ($v_z \rightarrow 1$). The interference between two scatterings is determined by the relative phase

factor

$$k(x_2 - x_1) = \omega t_2 - \vec{k}(\vec{x}_2 - \vec{x}_1) \approx L(\omega - k_{\parallel}) \equiv \frac{L}{\tau}, \quad (10)$$

with the formation time $\tau = 1/(\omega - k_{\parallel})$. Due to the dispersion relation eq. (1) the formation time is changed from the "vacuum" value $\tau_{\text{vac}}(k) = 2\text{ch}y/k_{\perp}$ [2] to $\tau(k, \omega_0) = 2\text{ch}y/\sqrt{k_{\perp}^2 + \omega_0^2}$ in a medium. In the region $k_{\perp} \ll \omega_0$ the reduction of the formation time due to the medium polarization can be considerable.

Multiple scattering: To estimate the effect connected with the reduction of the formation time in the medium we need to average over the interaction points \vec{x}_j in the general case of multiple scattering. Following the procedure developed in ref. [2] within the eikonal approximation one can perform the averaging of the phase factors by $\langle \exp\{ik(x_j - x_l)\} \rangle \approx \left(1 - \frac{i\lambda}{\tau(k, \omega_0)}\right)^{l-j}$ using a distribution of the length between successive scatterings, $L_j = z_{j+1} - z_j$, $dP = \lambda^{-1} \exp\{-L_j/\lambda\} dL_j$. As a result the spectrum for m -fold scattering can be expressed in the form

$$\frac{dn^{(m)}}{d^2k_{\perp} dy} = C_m(k, \omega_0) \frac{dn^{(1)}}{d^2k_{\perp} dy}, \quad (11)$$

where the radiation spectrum for the single scattering is defined by eq. (8) and the radiation formation factor is for not too small values of m

$$C_m(k, \omega_0) \approx m \frac{\chi^2}{1 + \chi^2} + \frac{1 - (1 - r_2)\chi^2}{r_2(1 + \chi^2)^2} \quad (12)$$

with $\chi(k, \omega_0) = \lambda/(r_2\tau(k, \omega_0))$ and $r_2 = C_A/(2C_2)$; $C_2 = C_A = N$ for gluons, $C_2 = C_F = (N^2 - 1)/(2N)$ for quarks. It should be stressed that, while the dependence of the radiation formation factor C_m on the parameter χ looks the same as in the vacuum case [2], this parameter itself is obviously modified by the reduction of the formation time in the medium. In fig. 4 the dependence of C_m on k_{\perp} is displayed for $m = 10$ for various values of the emitted gluon energy which defines the dimensionless parameter $a_0 = \lambda\omega_0^2/(2r_2\omega)$. As seen in fig. 4, due to the medium reduction of the formation time the radiation formation factor C_m , related to the LPM effect, is also modified, in particular in the region $k_{\perp} < \omega_0$.

Summary: In summary we have demonstrated the existence of a QCD analogue of the Ter-Mikaelian effect. In doing so a certain phase space region is selected where the treatment simplifies. The effect consists in the suppression of the induced radiation and the modification of the spectrum at low k_{\perp} due to the refractive properties of the medium.

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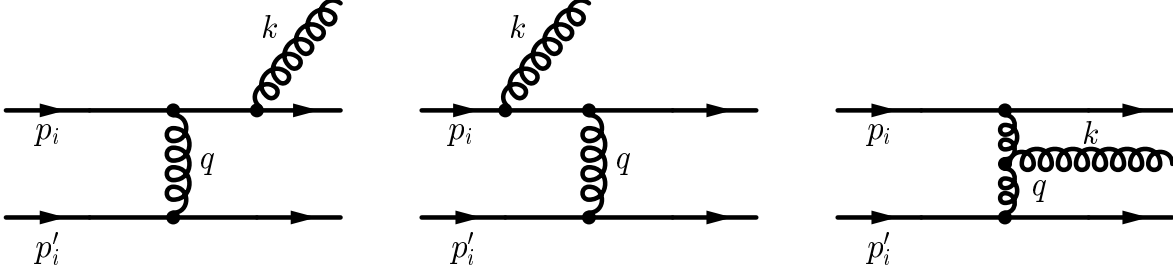


Figure 1: Feynman diagrams for single scattering which give the leading contribution to one-gluon emission in the $A^+ = 0$ gauge. The upper (lower) line is called projectile (target) quark line.

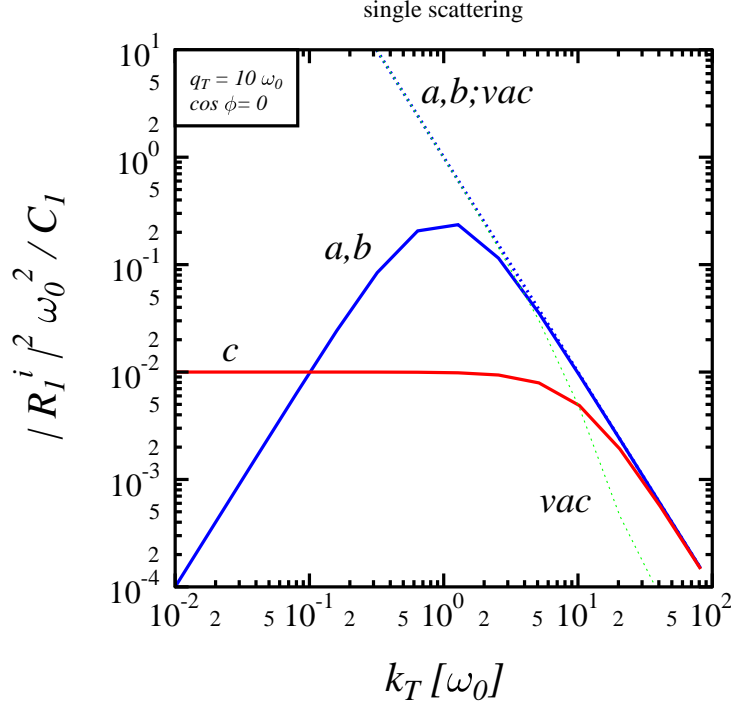


Figure 2: Various contributions $|R_1^i|^2$ to the radiation amplitude for single scattering as a function of k_\perp (in units of ω_0) for $q_\perp = 10\omega_0$. "a,b" ("a,b; vac") is for the radiation contribution from the quark line for $\omega_0 \neq 0$ (for vacuum, i.e. $\omega_0 = 0$), "c" is the contribution from the three-gluon vertex diagram, and "vac" labels the total amplitude in vacuum. To avoid complications with the collinear singularity at $\vec{k}_\perp \parallel \vec{q}_\perp$ we have chosen $\vec{k}_\perp \perp \vec{q}_\perp$.

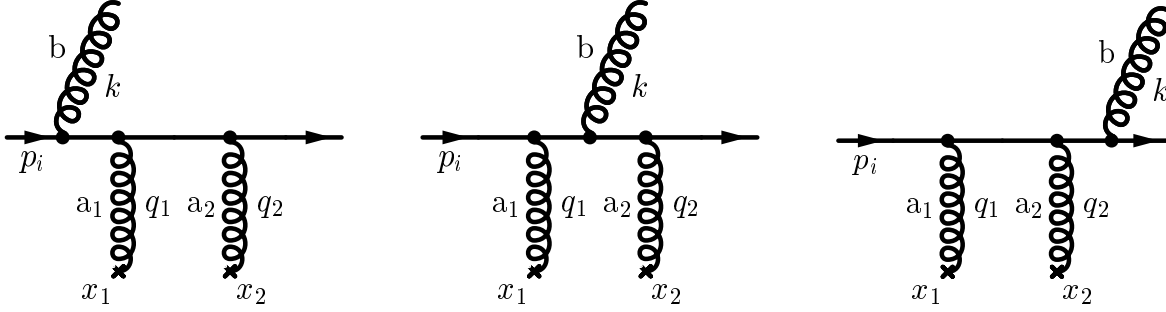


Figure 3: Relevant Feynman diagrams for double scattering at static centers.

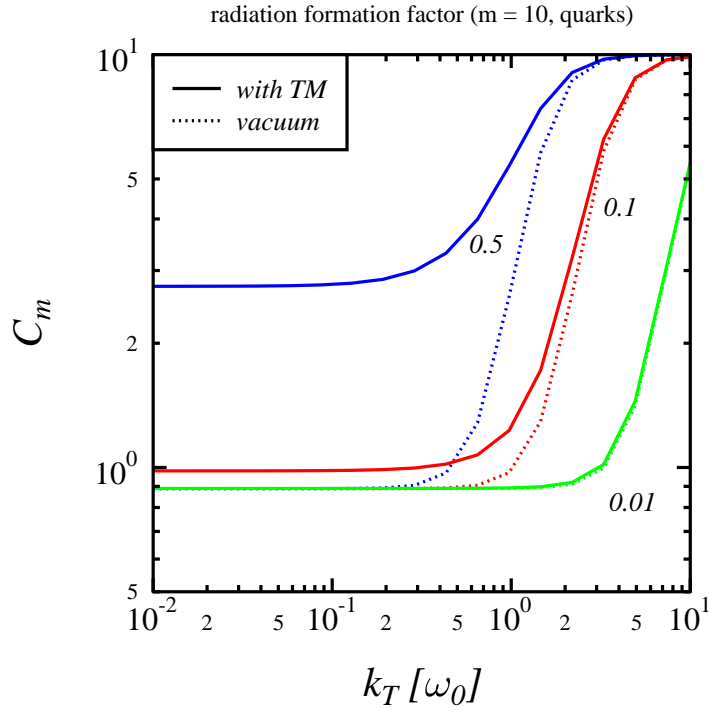


Figure 4: The radiation formation factor for quarks as a function of k_{\perp} (in units of ω_0) for $m = 10$. The curves are labeled by the value of a_0 ; full (dotted) curves: with (without) Ter-Mikaelian effect.